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EFFECTS OF VORTICITY, DISPLACEMENT SPEED AND CURVATURE ON HEAT TRANSFER WITH DISSIPATION

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Abstract—The method of matched asymptotic expansions has been used to analyze heat transfer with dissipation in forced convection, incompressible laminar flow on a surface with curvature. By using the well known solutions to the Falkner–Skan wedge flow and the corresponding energy equation with dissipation as the first order effects, the second order solutions to the energy equation including dissipation have been obtained. The calculated second order temperature profiles have been presented graphically as functions of the pressure gradient parameter and Prandtl number, to exhibit the effects of free-stream vorticity, displacement speed, longitudinal and transverse curvatures. The second order heat transfer, like the second order skin friction, becomes singular in adverse pressure gradients close to first order separation. Further, for the displacement speed problem and the curvature problems, other singularities occur at critical values of the pressure gradient parameter, just as for the second order skin friction. When there is no singularity, dissipation can affect heat transfer considerably, changing the Nusselt number from positive to negative, depending on the magnitude of the Eckert number. An increase of Prandtl number enhances the effects of dissipation.

NOMENCLATURE

a_i,	function specified in Table 1;
$A(\xi)$,	non-dimensionalized wall temperature distribution;
b_i,	function specified in Table 1;
$B(0)$,	a constant specifying free-stream vorticity;
C_1, C_2,	constants specifying first order velocity distribution and temperature at wall surface respectively;
C_p,	specific heat of fluid;
C_d, C_l, C_t,	constants specified in text;
D,	differential operator, $d/d\eta$;
E,	Eckert number;
$f(\eta)$,	non-dimensionalized first order stream function;
$g_i(\eta)$,	non-dimensionalized second order stream function;
$H(0)$,	a constant specifying free-stream enthalpy gradient;
$h_i(\eta)$,	non-dimensionalized second order temperature profile;
G_i, H_i, I_i,	functions specified in Table 1;
K,	thermal conductivity;
k,	local longitudinal surface curvature;
l,	characteristic length;
L_1, L_2, L_3,	operators as specified in the text;
m,	defined by the equation $m^2 = U_\infty^2/(C_p T_\infty)$;
n,	outer normal coordinate, non-dimensionalized;
N,	inner normal coordinate, non-dimensionalized;
Nu,	local Nusselt number, $h\bar{s}/K$;
P,	non-dimensional pressure in outer flow;
p,	non-dimensional pressure in inner flow;
Pr	Prandtl number, $\mu C_p/K$;
q_w,	local wall energy flux;
r,	transverse radius of curvature;
Re,	local Reynolds number, $U_{0s}\rho\bar{s}/\mu$;

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s, tangential non-dimensionalized coordinate $s = \bar{s}/l$;
$\bar{s}$, tangential coordinate;
t, non-dimensional temperature in inner flow;
T, non-dimensional temperature in outer flow;
T_w, specified wall surface temperature;
U_{0s}, inviscid surface speed to first order;
U, non-dimensional velocity in outer flow;
u, non-dimensional tangential velocity in inner flow;
V, non-dimensional normal velocity outer flow;
v, non-dimensional normal velocity in inner flow.

Greek symbols
α, limit of $(\eta - f)$ as $\eta \to \infty$;
β, pressure gradient parameter;
β_{sep}, value of β at separation, $-0{\cdot}198838$;
η, non-dimensionalized similarity variable;
ε, $(U_\infty \rho l/\mu)^{-\frac{1}{2}}$;
Φ, potential function in outer flow;
γ, wall temperature distribution parameter;
μ, absolute viscosity;
π_2, π_3, π_4, parameters specifying variation of displacement speed, longitudinal and transverse curvatures respectively;
ψ, Ψ, inner and outer stream functions respectively;
ρ, density of the fluid;
$\theta(\eta)$, non-dimensional first order temperature in similar flows;
ξ, variable defined in text.

Subscripts
c, complementary part;
p, particular part;
s, local quantity;
∞, free stream quantity.

Superscripts
v, vorticity;
d, displacement speed;
l, longitudinal curvature;
t, transverse curvature;
h, stagnation enthalpy.

1. INTRODUCTION

IN MANY problems of practical interest such as aerodynamic heating of bodies in flight, flow through rocket nozzles and flow over blades as in turbines and compressors, the calculation of heat transfer assumes great importance. In most of these problems, the surface over which the fluid flows is curved either longitudinally or transversely to the flow direction. For such problems, the application of the two dimensional boundary layer theory leads to the neglect of all longitudinal surface curvature effects, except for the dependence of the inviscid surface speed on such a curvature. Transverse curvature leads to equations of axisymmetric flow, which reduce to the two dimensional boundary-layer equations in Cartesian x, y-coordinates for incompressible steady flow through the use of the Mangler transformation [1].

The influence of longitudinal surface curvature on drag has been studied by several authors such as Murphy [2], Tani [3], Narasimha and Ojha [4] and Van Dyke [5]. Many of these authors have concerned themselves with jointly similar transformations [6] to obtain ordinary differential equations, which are finally solved on a digital computer. Werle and Davis [7] on the other hand, obtain solutions to the second order for incompressible flow over surfaces with longitudinal and transverse curvature, without the requirement of joint similarity. In obtaining these solutions, they have also taken into account the effects of free stream vorticity and the flow due to displacement speed. Three new parameters π_2, π_3 and π_4, one each for displacement speed, longitudinal curvature and transverse curvature are needed in addition to β, the pressure gradient parameter, to specify the solutions to the problem completely.

The effects of heat transfer in flow over solid surfaces have been investigated by Levy [8] and Schultz-Grunow and Breuer [9]. While Levy's solution is limited to an investigation of the boundary layer problem, Schultz-Grunow and Breuer's solution takes account of higher orders, though Gustafson and Pelech [10] have shown that there is a slight error in the analysis of [9] due to the use of the inviscid rather than the actual surface speed in the calculation. Moreover, both these solutions are limited to the investigation of heat transfer without dissipation. In two recent papers, Gupta and Kadambi [11] and Kadambi and Gupta [12] have investigated heat transfer from surfaces with longitudinal curvature, the first without dissipation, and the second with dissipation. It has been demonstrated [12] that dissipation can affect heat transfer considerably for fluids with high Prandtl numbers and cannot be neglected in all cases.

The present paper extends the analysis of Werle and Davis [7] to the study of heat transfer on surfaces with longitudinal and transverse curvature, including dissipation. It has been shown that similar solutions are obtainable up to the second order for all five cases listed by Van Dyke [14], namely (i) due to free stream vorticity, (ii) displacement speed (iii) longitudinal surface curvature, (iv) transverse surface curvature and (v) variation of free stream stagnation enthalpy. For these solutions to be valid, it is necessary as in other asymptotic developments of a similar kind, that the longitudinal and transverse curvatures be small. Further, for the class of similar solutions of the Falkner–Skan type which form the first order results in the asymptotic series development, it has been shown that the second order displacement speed is identically zero, when transverse curvature is of no importance. This result is identical to that for an infinite flat plate, and has been obtained by using the method of Kaplun [13]. The similarity equations for the second order problem depend on the parameters β, π_2, π_3, π_4, Prandtl number and Eckert number. These equations have been integrated numerically over the range of parameters $-0{\cdot}195 \leqslant \beta \leqslant 2$, $Pr = 0{\cdot}7$ and 10, and $-3{\cdot}0 \leqslant \pi_i \leqslant 2{\cdot}0$, ($i = 2, 3, 4$). The solutions to the heat-transfer problem have been limited to the case $\gamma = 2\beta$, so that computations may be minimized. Further, since the energy equations are linear to all orders, solutions have been obtained separately for the complementary and particular parts. The complete solution for any given Eckert number may be obtained from these separate solutions by superposition. The solutions have been graphically presented.

2. ANALYSIS

Consider a constant property incompressible fluid in steady flow past a curved surface. The effect of a wall temperature different from that of the fluid will be considered here in a manner similar to that of Kadambi and Gupta [12].

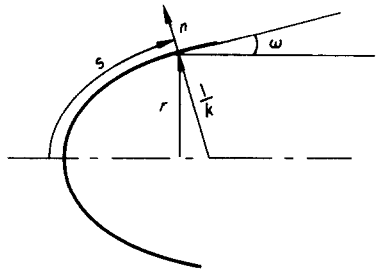

FIG. 1. Sketch showing coordinate system.

The coordinate system, (Fig. 1), the notation to be used and the procedure are identical to those of Werle and Davis [7], except for minor modifications. As is usual in these cases, one writes an outer expansion

$$\Psi_0(s, n; \varepsilon) \sim \Psi_1(s, n) + \varepsilon\Psi_2(s, n) + \dots \quad (1a)$$

$$U_0(s, n; \varepsilon) \sim U_1(s, n) + \varepsilon U_2(s, n) + \dots \quad (1b)$$

$$V_0(s, n; \varepsilon) \sim V_1(s, n) + \varepsilon V_2(s, n) + \dots \quad (1c)$$

$$P_0(s, n; \varepsilon) \sim P_1(s, n) + \varepsilon P_2(s, n) + \dots \quad (1d)$$

and

$$T_0(s, n; \varepsilon) \sim T_1(s, n) + \varepsilon T_2(s, n) + \dots \quad (1e)$$

where the subscript 0 denotes the inviscid outer flow for each of the quantities, the stream function Ψ, tangential and normal velocity components U, V, the pressure P and temperature T respectively, all of which have been suitably non-dimensionalized. The outer expansions yield, after substitution in the full Navier–Stokes and energy equations for an incompressible fluid and collecting terms of like powers of ε, solutions which are viscosity independent up to the second order. They can satisfy boundary conditions far upstream, and not those applicable near the wall.

The inner solutions are obtained by writing an inner expansion with an enlarged coordinate $N = n/\varepsilon$ ($\varepsilon = Re_\infty^{-\frac{1}{2}} \ll 1$) such that:

$$\psi_i(s, N; \varepsilon) \sim \varepsilon\psi_1(s, N) + \varepsilon^2\psi_2(s, N) + \ldots \quad (2a)$$

$$u_i(s, N; \varepsilon) \sim u_1(s, N) + \varepsilon u_2(s, N) + \ldots \quad (2b)$$

$$v_i(s, N; \varepsilon) \sim \varepsilon v_1(s, N) + \varepsilon^2 v_2(s, N) + \ldots \quad (2c)$$

$$p_i(s, N; \varepsilon) \sim p_1(s, N) + \varepsilon p_2(s, N) + \ldots \quad (2d)$$

and

$$T_i(s, N; \varepsilon) \sim t_1(s, N) + \varepsilon t_2(s, N) + \ldots \quad (2e)$$

the subscript i standing for the inner or viscous flow regime. The velocity components u_1, v_1 and u_2, v_2 are respectively related by the continuity relations

$$u_1 = \frac{1}{r^j}\frac{\partial\psi_1}{\partial N}; \quad v_1 = -\frac{1}{r^j}\frac{\partial\psi_1}{\partial s} \quad (3a)$$

$$u_2 + \left(\frac{j\cos\omega}{r}\right)Nu_1 = \frac{1}{r^j}\frac{\partial\psi_2}{\partial N} \quad (3b)$$

and

$$v_2 + \left(k + \frac{j\cos\omega}{r}\right)Nv_1 = -\frac{1}{r^j}\frac{\partial\psi_2}{\partial s}. \quad (3c)$$

In these equations, $j = 1$ for axisymmetric flow.

When the series are substituted in the full Navier–Stokes and energy equations and terms containing like powers of ε collected, the following sets of equations are obtained:

(a) First Order.

$$\psi_{1NNN} + \left(\psi_{1s}\frac{\partial}{\partial N} - \psi_{1N}\frac{\partial}{\partial s}\right)r^{-j}\psi_{1N} = -r^jU_1(s,0)\,U_{1s}(s,0) \quad (4a)$$

$$\frac{1}{Pr}t_{1NN} + \frac{1}{r^j}(\psi_{1s}t_{1N} - \psi_{1N}t_{1s}) + \frac{m^2}{r^{2j}}\psi_{1NN}^2 = 0. \quad (4b)$$

The initial and boundary conditions applicable to these equations obtained by matching are:

$$N = 0: \psi_1(s,0) = \psi_{1N}(s,0) = 0; t_1(s,0) = T_w(s)/T_\infty \quad (5a)$$

$$N \to \infty: \psi_{1N}(s,N) \to r^jU_1(s,0); t_1(s,N) \to T_1(s,0)/T_\infty. \quad (5b)$$

(b) Second Order.

$$\psi_{2NNN} + \left(\psi_{1s}\frac{\partial}{\partial N} - \psi_{1N}\frac{\partial}{\partial s}\right)r^{-j}\psi_{2N} + \left(\psi_{2s}\frac{\partial}{\partial N} - \psi_{2N}\frac{\partial}{\partial s}\right)r^{-j}\psi_{1N} = r^j\frac{\partial}{\partial s}$$
$$\{k[NU_1^2(s,0) + \int_N^\infty (U_1^2(s,0) - r^{-2j}\psi_{1N}^2)\mathrm{d}N]\}$$
$$- k[N\psi_{1NNN} + \psi_{1NN} + r^{-j}\psi_{1N}\psi_{1s}]$$
$$- r^{2j}B(0)\,V_2(s,0) - r^j\frac{\mathrm{d}}{\mathrm{d}s}[U_1(s,0)\,U_2(s,0)]$$
$$+ \frac{j\cos\omega}{r}\left[\psi_{1NN} + \left(\psi_{1s}\frac{\partial}{\partial N} - \psi_{1N}\frac{\partial}{\partial s}\right)\frac{N\psi_{1N}}{r^j} - Nr^jU_1(s,0)\,U_2(s,0)\right] \quad (6a)$$

$$\frac{1}{Pr}t_{2NN} - \frac{1}{r^j}(\psi_{1N}t_{2s} - \psi_{1s}t_{2N}) = -k\left[\frac{\partial}{\partial N}\left(\frac{Nt_{1N}}{Pr}\right) + m^2r^{-2j}(N\psi_{1NN}^2 - 2\psi_{1N}\psi_{1NN})\right)$$
$$-\frac{j\cos\omega}{r}\left[\frac{\partial}{\partial N}\left(\frac{Nt_{1N}}{Pr}\right) - \frac{m^2}{r^{2j}}(N\psi_{1NN}^2\right.$$

$$+ 2\psi_{1N}\psi_{1NN})\Big] - 2m^2 r^{-2j}\psi_{1NN}\psi_{2NN} + \frac{1}{r^j}(\psi_{2N}t_{1s} - \psi_{2s}t_{1N}). \quad (6b)$$

The initial and boundary conditions applicable to this set, obtained by matching to the proper order are:

$$N = 0: \psi_2(s,0) = \psi_{2N}(s,0) = 0; t_2(s,0) = 0 \quad (7a)$$

$$N \to \infty: \psi_{2N}(s,N) \to Nr^j\left[\left(\frac{j\cos\omega}{r} - k\right)U_1(s,0) + r^j B(0) + r^j U_2(s,0); t_2(s,N) \to H(0)\left[Nr^j U_1(s,0) + \Psi_2(s,0)\right]. \right. \quad (7b)$$

One now splits $\psi_2(s,N)$ and $t_2(s,N)$ into various parts as given by the equations:

$$\psi_2 = B(0)\psi_2^v + U_2(s,0)\psi_2^d + k\psi_2^l + \frac{j\cos\omega}{r}\psi_2^t \quad (8a)$$

$$t_2 = B(0)t_2^v + U_2(s,0)t_2^d + kt_2^l + \frac{j\cos\omega}{r}t_2^t + H(0)t_2^h \quad (8b)$$

where $\psi_2^v, \psi_2^d, \ldots, t_2^h$ are functions which are determinable from the differential equation. They can be superposed appropriately to yield ψ_2 and t_2, due to the linearity of the system under consideration.

Similarity transformations. The similarity variables to be used for the first order system are well-known, being given by:

$$\xi = \int_0^s U_1 r^{2j}\,ds, \quad \eta = \frac{U_1 r^j}{\sqrt{(2\xi)}}N \quad (9a)$$

$$\psi_1 = \sqrt{(2\xi)}f(\eta) \quad \text{and} \quad \theta(\eta) = \frac{t_1 - (T_1/T_\infty)}{(T_w/T_\infty - T_1/T_\infty)} \quad (9b)$$

where $U_1 = U_1(s,0)$ and $(T_w - T_1)/T_\infty = A$, both U_1 and A being in general, arbitrary functions of s. In order to obtain similarity however, one requires that U_1 and A should be power functions of ξ where

$$U_1 = C_1\xi^{\beta/2} \quad \text{and} \quad A = C_2\xi^{\gamma/2},$$

β and γ being constants. The constant β is the well known Falkner–Skan pressure gradient parameter, with values in the range $-0{\cdot}198838 \leqslant \beta \leqslant 2{\cdot}0$, while γ is arbitrary.

If the quantities above are all substituted in the first order equations (4a) and (4b), it is easily seen that they reduce to the forms

$$f''' + ff'' + \beta(1 - f'^2) = 0 \quad (10a)$$

$$\theta'' + Pr[f\theta' - 2\beta f'\theta + Ef''^2] = 0 \quad (10b)$$

where E is the Eckert number. As seen from [12], the equations reduce to the above forms only if $\gamma = 2\beta$.

If dissipation is neglected, it is possible to leave γ arbitrary and obtain the energy equation in the form:

$$\theta'' + Pr[f\theta' - \gamma f'\theta] = 0. \quad (10c)$$

The initial and boundary conditions are:

$$\eta = 0: f(0) = f'(0) = 0; \quad \theta(0) = 1 \quad (11a)$$

$$\eta \to \infty: f'(\eta) \to 1; \quad \theta(\eta) \to 0. \quad (11b)$$

The term $Ef''^2(\eta)$ is usually negligible for fluids with small Prandtl numbers and also, for gases where compressibility effects are negligible. The term is not negligible for viscous liquids, whose Prandtl numbers are in the range of 10–100 or more.

The second order problem is amenable to a similarity analysis through the use of the transformations

(a) Vorticity effect

$$\psi_2^v = \frac{2\xi}{U_1^2}g_1(\eta); \quad t_2^v = \frac{A\sqrt{(2\xi)}}{U_1^2}h_1(\eta) \quad (12a)$$

(b) Displacement speed effect

$$\psi_2^d = \frac{\sqrt{(2\xi)}}{U_1}g_2(\eta); \quad t_2^d = \frac{A}{U_1}h_2(\eta) \quad (12b)$$

(c) Effect of longitudinal curvature

$$\psi_2^l = \frac{2\xi}{r^j U_1} g_3(\eta); \quad t_2^l = \frac{A\sqrt{(2\xi)}}{r^j U_1} h_3(\eta) \quad (12c)$$

(d) Effect of transverse curvature

$$\psi_2^t = \frac{2\xi}{r^j U_1} g_4(\eta); \quad t_2^t = \frac{A\sqrt{(2\xi)}}{r^j U_1} h_4(\eta) \quad (12d)$$

(e) Effect of free-stream stagnation enthalpy

$$\psi_2^h = 0; \quad t_2^h = \sqrt{(2\xi)} h_5(\eta). \quad (12e)$$

The second order problems generated through the use of the above equations will be written in the forms:

$$L_1(g_i) = G_i \quad (13a)$$

$$L_2(h_i) = L_3(g_i) + H_{ic} + Pr\, E\,(H_{ip} - 2f''g_i'') \quad (13b)$$

where L_1, L_2 and L_3 are operators given by the equations

$$L_1 = D^3 + fD^2 - a_i f'D + b_i f'' \quad (14a)$$

$$L_2 = D^2 + Pr\,(fD - a_i f') \quad (14b)$$

and

$$L_3 = Pr\,(2\beta\theta D - b_i\theta') \quad (14c)$$

where the subscript i takes integral values in the range 1–5.

The boundary conditions applicable to this set are:

$$\eta = 0: \quad g_i(0) = g_i'(0) = 0; \quad h_i(0) = 0 \quad (15a)$$

$$\eta \to \infty: \quad g_i'(\eta) \to F_i(\eta); \quad h_i(\eta) \to I_i(\eta). \quad (15b)$$

The quantities a_i, b_i, G_i, F_i, H_{ic} and H_{id} are all exhibited in Table 1. The functions are now seen to depend on three new parameters, π_2 for displacement speed, π_3 for longitudinal curvature and π_4 for transverse curvature. In general, these constants are arbitrary. However, if one desires to obtain jointly similar solutions, the constants are related to β by the equations:

$$\pi_2 = \beta, \pi_3 = \beta - 1 \text{ and } \pi_4 = \beta - 1.$$

If these conditions for joint similarity are satisfied, the quantities a_i, b_i and G_2, G_3, G_4 simplify as given by the substitutions:

$$a_i = 2\beta, b_i = 1 \quad (16a)$$

$$G_2 = -2\beta, G_3 = \eta\left[\frac{2\beta^2}{1+\beta} - f'''\right] + \frac{\beta - 1}{\beta + 1}(f'' + ff') + \frac{2\alpha\beta}{1+\beta} \quad (16b)$$

and

$$G_4 = \eta[ff'' - \beta(f'^2 + 1)] + f'' + ff'. \quad (16c)$$

3. SOLUTIONS OF THE DIFFERENTIAL EQUATIONS

The differential equations exhibited earlier contain the parameters β, π_2, π_3, π_4, the Prandtl and Eckert numbers. Of these parameters, the Eckert number appears only in the non-homogeneous part of the energy equation. Hence, the problem of solving the full set of equations for parametric variations in Eckert number may be avoided by writing

$$\theta = \theta_c + E\theta_p \text{ and } h_i = h_{ic} + Eh_{ip}, i = 1\text{–}5 \quad (17)$$

where the subscript c denotes the complementary part and the subscript p, the particular part of the solution to the differential equation. Then, the equations for the complementary and particular parts may be separately written as:

$$\theta_c'' + Pr[f\theta_c' - 2\beta f'\theta_c] = 0 \quad (18a)$$

$$\theta_p'' + Pr[f\theta_p' - 2\beta f'\theta_p + f''^2] = 0 \quad (18b)$$

and,

$$L_2(h_{ic}) = H_{ic} + L_{3c}(g_i) \quad (19a)$$

$$L_2(h_{ip}) = H_{ip} - 2Pr f''g_i'' + L_{3p}(g_i) \quad (19b)$$

where

$H_{ic} = -(\eta\theta_c'' + \theta_c')$ and $H_{ip} = -Pr(\eta f''^2 - 2f'f) - (\eta\theta_p'' + \theta_p')$ if $i = 3$ and $H_{ip} = Pr \times (\eta f''^2 + 2f'f'') - (\eta\theta_p'' + \theta_p')$ if $i = 4$.

For all other values of i, both H_{ic} and H_{ip} are

Table 1

i	Parameters	a_i	b_i	G_i	F_i	H_{ic}	H_{ip}	$I_i(\eta)$
1	Vorticity	1	$2(1-\beta)$	$-\alpha$	η	0	0	0
2	Displacement: $\pi_2 = \frac{2\xi}{U_2}\frac{dU_2}{d\xi}$	$\pi_2 + \beta$	$1 + \pi_2 - \beta$	$-(\pi_2 + \beta)$	1	0	0	0
3	Longitudinal curv: $\pi_3 = \frac{2\xi}{(k/r^j)}\frac{d(k/r^j)}{d\xi}$	$1 + \pi_3 + \beta$	$2 + \pi_3 - \beta$	$\eta\left[\beta\left(1 + \frac{\pi_3}{1+\beta}\right) - f'''\right] + \frac{\pi_3}{1+\beta}(f'' + ff') + \left(1 + \frac{\pi_3}{1+\beta}\right)\alpha$	$-\eta$	$-(\eta\theta'' + \theta')$	$-(\eta f''^2 - 2f'f'')$	0
4	Trans. curv: $\pi_4 = \frac{2\xi}{(\cos\omega/r^{2j})}\frac{d(\cos\omega/r^{2j})}{d\xi}$	$1 + \pi_4 + \beta$	$2 + \pi_4 - \beta$	$\eta[ff'' - (1 + \pi_4)f'^2 - \beta] + f'' + ff'$	η	$-(\eta\theta'' + \theta')$	$(\eta f''^2 + 2f'f'')$	0
5	Stagnation enthalpy	1	$1 - \beta$	0	0	0	0	$(\eta - \alpha)$

zero. The boundary conditions pertaining to this set are:

$$\eta = 0: \theta_c(0) = 1; \theta_p(0) = 0; h_{ic}(0) = h_{ip}(0) = 0$$
$$(\text{for all } i = 1,4); h_5(0) = 0 \quad (20a)$$

$$\eta \to \infty: \theta_c(\eta) \to 0; \theta_p(\eta) \to 0; h_{ic}(\eta) \to h_{ip}(\eta) \to 0;$$
$$(i = 1,4); h_5(\eta) \to (\eta - \alpha). \quad (20b)$$

There does not arise any necessity to split $h_5(\eta)$ into its complementary and particular parts, since the equation for $h_5(\eta)$ is homogeneous, and independent of Eckert number. This implies that the effect of free-stream enthalpy gradient is unaltered by dissipation for an incompressible fluid.

If one obtains solutions to all the equations in the above form, the homogeneous and non-homogeneous parts may be superposed for any specified value of Eckert number, and a composite solution determined therefrom, due to the linear character of the equations being studied. The necessity for solving the equations for various Eckert numbers is thereby eliminated. Even so, there are five parameters β, π_2, π_3, π_4 and Pr which affect the equations parametrically.

Though the effect of displacement speed on the solutions is of great interest, the determination of $U_2(s, 0)$ presents great difficulties in many cases, and hence very few problems have been solved where $U_2(s, 0)$ is completely known. For the infinite flat plate and the Falkner–Skan wedge problem, $U_2(s, 0)$ has been calculated by Kaplun [13], while Van Dyke [14] has calculated it for flow over a parabola with a constant free-stream speed. It will now be shown that $U_2(s, 0)$ can be completely calculated for flow over a surface with only longitudinal curvature and no transverse curvature. The procedure adopted here is similar to that of Kaplun [13].

Consider the first-order problem posed by equations (9a), (9b), (10a) and (10b), with no transverse curvature, i.e. $j = 0$. Let Ψ_1 and Φ_1 be the first order outer stream function and potential function respectively. Then, by the definition of the potential and stream functions, one obtains:

$$\frac{1}{1 + nk}\frac{\partial \Phi_1}{\partial s} = U_1; \quad \frac{\partial \Phi_1}{\partial n} = V_1 \quad (21a)$$

$$\frac{\partial \Psi_1}{\partial n} = U_1 \text{ and } \frac{\partial \Psi_1}{\partial s} = -(1 + nk) V_1 \quad (21b)$$

where U_1 and V_1 are the first order outer velocities in the s and n-directions respectively, as as specified by equations (1b) and (1c). With these definitions, it is seen that the Laplacian ∇^2 expressed in terms of Φ_1 and Ψ_1 as independent coordinates assumes the form:

$$\nabla^2 \equiv (1 + nk)(U_1^2 + V_1^2)\left[\frac{\partial^2}{\partial \Phi_1^2} + \frac{\partial^2}{\partial \Psi_1^2}\right]. \quad (22)$$

Further, if one transforms the coordinates from (s, n) to (Φ_1, Ψ_1) coordinates and solves the boundary-layer problem (first order) in the new coordinates, the new solution can be expressed in terms of (Φ_1, Ψ_1) by using Kaplun's correlation theorem [13] in the form:

$$\psi_1 = \sqrt{[2\xi(\Phi_1, 0]} f\left\{\frac{(\sqrt{Re})\, U_1 \Psi_1}{\sqrt{[2\xi(\Phi_1, 0)]}}\right\}.$$

However, from equation (9a) it is clear that

$$\Phi_1(s, 0) = \int_0^s U_1(s, 0)\, ds = \xi.$$

Also, at $n = 0$, $\Psi_1 = 0$. These two requirements together lead to the result:

$$\psi_1 = \sqrt{(2\Phi_1)} f\left[\frac{\sqrt{Re}\, U_1 \Psi_1}{\sqrt{(2\Phi_1)}}\right] \quad (23)$$

in terms of the new coordinates Φ_1 and Ψ_1. This expression may now be used to find the value of $\Psi_2(\Phi_1, 0)$ by using the matching principle, when it is seen that

$$\Psi_2(\Phi_1, 0) = -\alpha\sqrt{(2\Phi_1)}, \alpha = \lim_{\eta \to \infty} [\eta - f(\eta)].$$

Further, it is required that $\Psi_2(\Phi_1, 0) = 0(\Psi_1)$ far upstream. Since Ψ_2 should satisfy the Laplace equation, $\nabla^2 \Psi_2 = (\partial^2/\partial \Phi_1^2 + \partial^2/\partial \Psi_1^2)\, \Psi_2 = 0$

and hence,

$$\Psi_2(\Phi_1, \Psi_1) = -\alpha Re[\Phi_1 + i\Psi_1]^{\frac{1}{2}} \qquad (24)$$

where $Re[\ ]$ represents the real part of the complex function in the square brackets. It is readily recognized that this result is identical to that of Kaplun, except that the present result applies even to surfaces with longitudinal curvature. This equation enables one to compute

$$U_2(s,0) = \left[\frac{\partial \Psi_2}{\partial n}(\Phi_1, \Psi_1)\right]_{n=0}$$
$$= \frac{\alpha}{2} Re\left\{\frac{1}{(\Phi_1 + i\Psi_1)^{\frac{1}{2}}}\left[\frac{\partial \Phi_1}{\partial n} + i\frac{\partial \Psi_1}{\partial n}\right]\right\}_{n=0}$$
$$= \frac{\alpha}{2} Re\left\{\frac{1}{\Phi_1^{\frac{1}{2}}}(V_1 + iU_1)\right\}_{n=0} = 0.$$

The second order velocity $U_2(s,0)$ vanishes identically at the surface of a body satisfying the conditions specified earlier, just as for an infinite flat plate. This result does not apply to a finite body since the effect of the trailing edge is felt upstream, and solutions of the kind assumed in the above analysis do not hold at the trailing edge itself. Further, for a body with transverse curvature, $U_2(s,0)$ cannot be computed as above, since the Laplacian does not reduce to the form $(\partial^2/\partial\Phi_1^2 + \partial^2/\partial\Psi_1^2)$.

From the above computations, it is clear that $U_2(s,0)$ cannot be arbitrarily prescribed in general. To state it differently, $U_2(s,0)$ can at least in principle, be determined completely as soon as the first order outer and inner flows are specified so that one cannot require that U_2 be specified by the expression $(2\xi/U_2)(dU_2/d\xi) = \pi_2$, after having initially specified $U_1(s,0)$ and the inner stream function Ψ_1. Hence, in general, the solutions obtained by using the extra requirement specified above on U_2 cannot be expected to represent solutions to the same problem for which the first order solutions have been obtained. In spite of this, following Werle and Davis [7], computations have been carried out for the second order equations for various parametric values of π_2, since the solutions exhibit certain important characteristics of the second order equations.

Numerical solutions have been obtained for cases (i) external free-stream vorticity, (ii) effect of displacement thickness, (iii) effect of longitudinal curvature and (iv) effect of transverse curvature to determine the variation in second order heat transfer due to each of these. No numerical solutions relating to the effect of free-stream stagnation temperature gradient have been attempted, since in practice an incompressible fluid with high Prandtl number is most unlikely to have a stagnation enthalpy gradient large enough to cause observable effects on heat transfer. The parameters β, π_2, π_3, and π_4 have been allowed to take on values in the ranges $-0{\cdot}195 \leqslant \beta \leqslant 2{\cdot}0$, $-3{\cdot}0 \leqslant \pi_i \leqslant 2{\cdot}0$, $i = 2, 3, 4$. These ranges are the same as those for which Werle and Davis have obtained numerical solutions to the second order momentum equation alone. The energy equation has been solved for two specific Prandtl numbers, $Pr = 0{\cdot}7$ and $10{\cdot}0$, to bring out the effects of increasing Prandtl numbers on dissipation. An attempt to obtain solutions with Prandtl number 100 was abandoned after a few runs on the computer because of the requirements of extremely small step size (0·001 or lower in some cases), and the inordinately long computer time involved.

Table 2. Comparison between present computations and those of [7]

β	$g_1''(0)$ Present	$g_1''(0)$ Ref. [7]
−0·195	19·465235	19·4652
−0·18	9·256877	9·2569
−0·16	6·632903	6·6329
−0·10	4·315440	4·3154
−0·05	3·572876	—
0·0	3·125983	3·1260
0·20	2·271988	2·2720
0·50	1·768595	1·7686
1·0	1·406544	1·4065
1·6	1·197881	1·1979
2·0	1·1106	1·1106

All the numerical solutions were obtained using a Runge–Kutta–Gill algorithm, with step sizes in the range 0·025–0·05, using single precision arithmetic on a CDC 6400 digital computer at Lehigh University. Double precision computations were not attempted since the CDC 6400 computer uses 14 significant digits in single precision, and this was thought to be sufficient for all computations. The numerical solutions compute the initial values $g_i''(0)$, $h_{ic}'(0)$ and $h_{ip}(0)$ ($i = 1, 2, 3, 4$). In order to check the accuracy of the present calculations, Table 2 has been prepared comparing the newly computed $g_1''(0)$ with those of Werle and Davis [7]. The agreement is seen to be excellent including the fourth decimal place. The accuracy of the calculations on displacement effect was checked by solving the equations for $g_2''(0)$ when $\pi_2 = \beta$. For this particular case, [7] provides an exact solution which requires that $g_2''(0) = 1{\cdot}5f''(0)$. The difference $g_2''(0) - 1{\cdot}5f''(0)$ was computed for all β in the range of calculations, and the results were found to be zero up to six decimal places. The calculations for $g_3''(0)$ and $h_3'(0)$ were checked against the results of Kadambi and Gupta [12] for the case $\pi_3 = \beta - 1$, and exact agreement was found to six decimal places. Since the values of $g_4''(0)$ presented in [7] are graphical it has not been possible to compare the presently computed values with those of the earlier ones to a sufficient number of significant digits. Nevertheless, it is felt that the present calculations are quite accurate, since $g_1''(0)$ and $g_3''(0)$ have all been determined accurately.

Having obtained the unknown derivatives $h_{1c}'(0)$ and $h_{1p}'(0)$, one can write an expression for the local energy flux at the wall in the form:

$$q_w = -\frac{K}{l}\left(\frac{\partial T}{\partial n}\right)_{n=0} = -\frac{K}{l\varepsilon}\left(\frac{\partial T}{\partial N}\right)_{N=0}.$$

On substituting for T and simplifying, the local heat transfer coefficient is found to be given by the expression

$$\frac{h\tilde{s}}{K} = Nu = -\left(\frac{U_1 s r^{2j}}{2\xi}\right)^{\frac{1}{2}} (Re_s)^{\frac{1}{2}} \left\{\theta'(0) + \frac{\varepsilon\sqrt{(2\xi)}}{U_1}\right.$$

$$\left.\left[\frac{B(0)}{U_1} h_1'(0) + \frac{U_2}{\sqrt{(2\xi)}} h_2'(0) + \frac{k}{r^j} h_3'(0) + \frac{j\cos\omega}{r^{2j}} h_4'(0)\right]\right\}. \quad (25)$$

The change in the local Nusselt number due to the second order effect is given by the equation:

$$(\Delta Nu)(Re)^{-\frac{1}{2}} = -\left(\frac{sr^{2j}}{U_1}\right)^{\frac{1}{2}} \left[\frac{B(0)}{U_1} h_1'(0) + \frac{U_2}{\sqrt{(2\xi)}} h_2'(0) + \frac{k}{r^j} h_3'(0) + \frac{j\cos\omega}{r^{2j}} h_4'(0)\right]. \quad (26)$$

4. RESULTS AND DISCUSSION

All the previous equations are for the case of flow including dissipation, when $\gamma = 2\beta$ for similarity. The solutions obtained through numerical calculations have been presented in Figs. 2–10.

The effects of external stream vorticity on the second order heat transfer are shown in Figs. 2–4. Figure 2 shows the second order function $h_{1c}(\eta)$ with β as a parameter and $Pr = 10$. For all β, $h_{1c}(\eta)$ starts from zero and becomes negative before tending to zero for sufficiently large η. Also, as one may expect, a change in β from positive to negative values increasing towards separation causes $h_{1c}(\eta)$ to increase in magnitude for any η. This trend is similar to that of $g_1''(\eta)$ noticed by Werle and Davis [7], and shows that the asymptotic expansions may not be uniformly valid for large adverse pressure gradients. For small β of course, the effect of free stream vorticity is small, and the solutions are uniformly valid up to the second order.

Figure 3 shows $h_{1p}(\eta)$ with β as a parameter, again for $Pr = 10$. Unlike $h_{1c}(\eta)$, $h_{1p}(\eta)$ starts being positive and changes to negative values before tending to zero for large η. Again, changing the pressure gradient from favourable to adverse increases h_{1p} in magnitude. Further, $h_{1p}(\eta)$ and $h_{1c}(\eta)$ are of opposite signs at the wall so that dissipation tends to reduce the wall temperature gradient and hence heat transfer, even to the second order. The actual second order

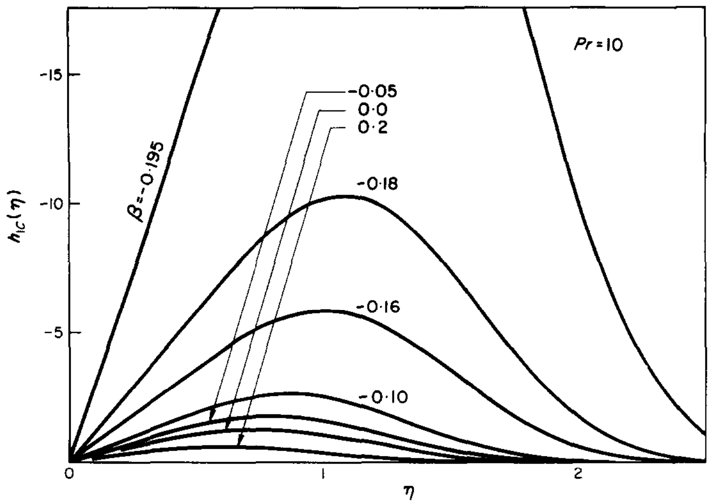

FIG. 2. Temperature profiles $h_{1c}(\eta)$ with β as a parameter.

profile may be obtained by superposing h_{1c} and h_{1p} by the relation $h_1(\eta) = h_{1c}(\eta) + Eh_{1p}(\eta)$.

The second order temperature gradients $h'_{1c}(0)$ and $h'_{1p}(0)$ are exhibited in Fig. 4 as functions of β, for $Pr = 0{\cdot}7$ and 10. When the Prandtl number is small, both the first and second order gradients are small and increase with increasing Prandtl numbers. The second order gradients become singular for β close to separation as given by the approximate relations

$$h'_{1c}(0) \propto (\beta - \beta_{sep})^{-0.545}, \quad Pr = 0{\cdot}7$$

$$h'_{1c}(0) \propto (\beta - \beta_{sep})^{-0.6}, \quad Pr = 10.$$

The gradients $h'_{1p}(0)$ seem to behave similar to $h'_{1c}(0)$ for the differing Prandtl numbers. These

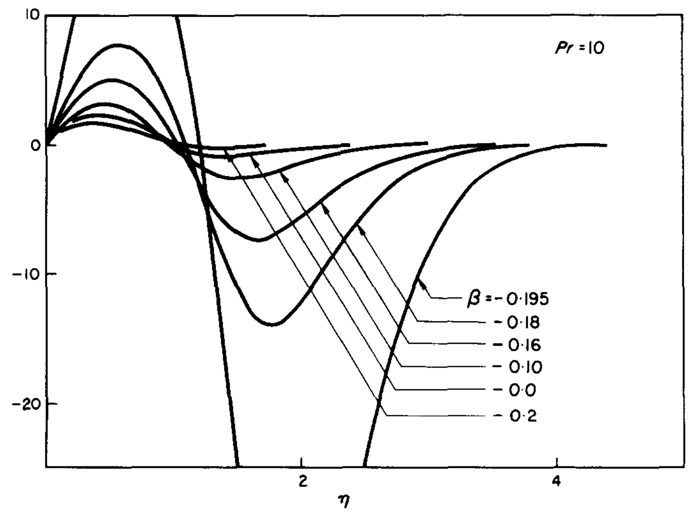

FIG. 3. Temperature profiles $h_{1p}(\eta)$ with β as a parameter.

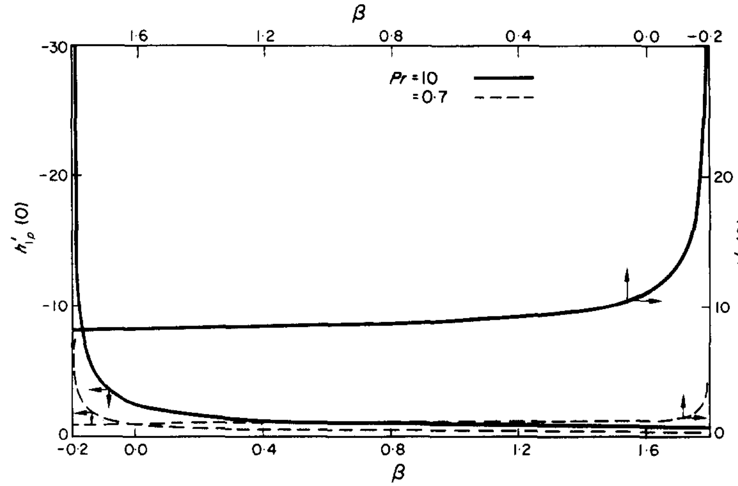

FIG. 4. Plot of wall temperature gradients $h'_{1c}(0)$ and $h'_{1p}(0)$ against β.

relations are similar to those given by [7] for $g_1''(0)$, except that presently, there is a dependence on Prandtl number, as may be expected. Since $h'_{1c}(0)$ and $h'_{1p}(0)$ are of opposite signs, an increasing Eckert number changes $h'_1(0)$ from negative to positive. For large Eckert numbers, the heat transfer occurs therefore from the fluid to the wall, even if the free stream temperature is lower than that at the wall.

Figure 5 shows the effect of displacement speed on the wall temperature gradient $h'_{2c}(0)$ for various π_2 and Prandtl numbers. These curves are similar to those of $h'_{1c}(0)$ since for all positive π_2, $h'_{2c}(0)$ is negative and becomes nearly constant for large favourable pressure gradients. For adverse pressure gradients, $h'_{2c}(0)$ becomes singular near β_{sep}. For negative values of π_2 however, it is possible for $h'_{2c}(0)$ to become positive in a small range of values of β in the neighbourhood of zero. Further, singularities appear in the second order functions for negative π_2, when $\beta > \beta_{sep}$. This observation is similar to that of Werle and Davis [7], who showed the existence of these singularities in the second order shear stresses at various critical values of π_2. There are only two values of π_2, namely $\pi_2 = \beta_{sep}$ and $\pi_2 = 2(1 - \beta_{sep})$ for which the first order separation point β_{sep} does not represent a singularity for the second order solutions. To demonstrate that there is no singularity in $h'_{2c}(0)$ at $\pi_2 = \beta_{sep}$, the values of $h'_{2c}(0)$ for various β have been plotted in Fig. 5 for the jointly similar family of solutions $\pi_2 = \beta$. It is seen that this curve does not exhibit the steep rise characteristic of the other plots near $\beta = \beta_{sep}$. For positive π_2, the temperature gradients $h'_{2c}(0)$ which have singularities at $\beta = \beta_{sep}$ behave approximately

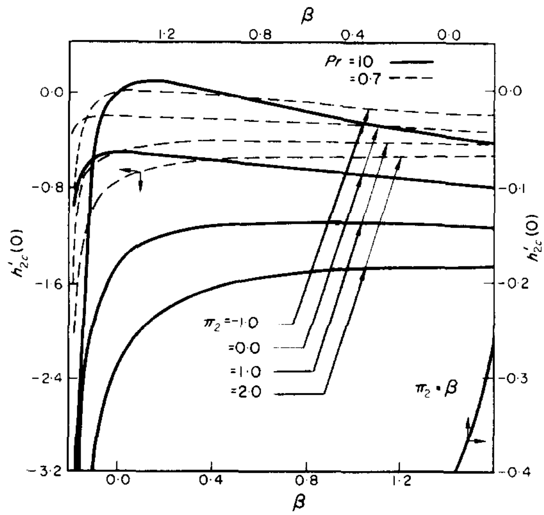

FIG. 5. Plot of temperature gradients $h'_{2c}(0)$ against β for $Pr = 0{\cdot}7$ and 10.

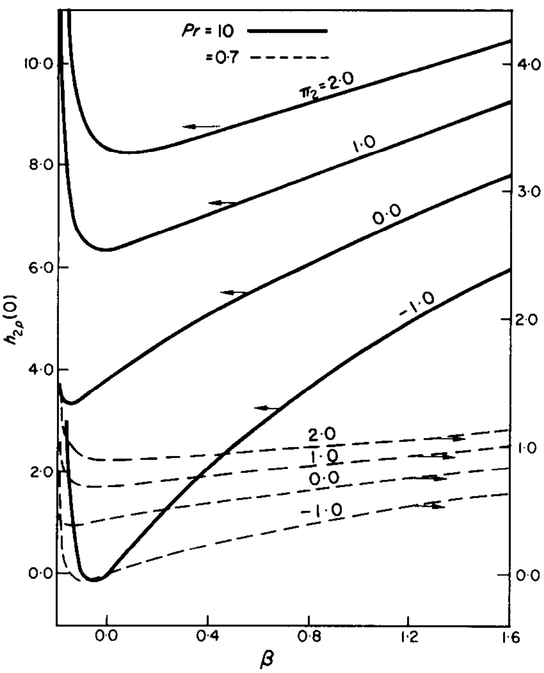

FIG. 6. Plot of temperature gradients $h'_{2p}(0)$ against β for $Pr = 0{\cdot}7$ and 10.

according to the equations

$$h'_{2c}(0) \propto (\beta - \beta_{sep})^{-0{\cdot}48}, \quad Pr = 0{\cdot}7$$

$$h'_{2c}(0) \propto (\beta - \beta_{sep})^{-0{\cdot}525}, \quad Pr = 10.$$

The non-homogeneous part $h'_{2p}(0)$ is positive for all π_2 in the range $-1{\cdot}0 \leqslant \pi_2 \leqslant 2{\cdot}0$. For still larger negative π_2, there appear the usual singularities in the system and the curves corresponding to such π_2 have not been plotted. Just as for the vorticity effect, dissipation tends to make the wall temperature gradient positive for sufficiently large Eckert numbers. Again, there is no singularity at $\beta = \beta_{sep}$ when $\pi_2 = \beta_{sep}$ and when $\pi_2 = 2(1 - \beta_{sep})$.

The effect of longitudinal surface curvature on second order temperature gradients is exhibited in Figs. 7 and 8. Figure 7 is drawn for $Pr = 0{\cdot}7$ and Fig. 8 for $Pr = 10$. Both the wall gradients $h'_{3c}(0)$ and $h'_{3p}(0)$ are relatively small in magnitude for favorable pressure gradients and become large close to separation. The singularities close to separation in the range of positive π_3 may be approximated by the equation

$$h'_{3c}(0) \propto (\beta - \beta_{sep})^{-0{\cdot}58}, \quad Pr = 0{\cdot}7$$

$$h'_{3c}(0) \propto (\beta - \beta_{sep})^{-0{\cdot}6}, \quad Pr = 10.$$

An interesting aspect of these solutions is that for $\pi_3 > -1{\cdot}0$, the non-dimensionalized second order temperature gradient $h'_{3c}(0)$ (without dissipation) is positive unlike $h'_{1c}(0)$ and $h'_{2c}(0)$. Hence, for convex curvature, (k/r^j positive), the second order effect is opposed to the first order effect without dissipation, while for concave curvature (k/r^j negative), the second and first order heat transfers without dissipation are of the same sign. This observation is in conformity with the results of [12], which were obtained for jointly similar solutions. On the other hand, the second order effect of dissipation is to enhance heat transfer from the wall to the fluid for convex curvature and oppose it for concave curvature. If the Eckert number is large, dissipation causes increased heat transfer from the wall to the fluid when the curvature changes from concave to convex. If the Eckert number is small, increasing the concavity of the curvature increases heat transfer, but in the direction from the fluid to the wall. This effect is enhanced by increasing Prandtl numbers as well. The change in heat transfer due to curvature is rather small at small Eckert numbers (about 8–10 per cent), while for Eckert numbers larger than about 0·5, the change in heat transfer due to curvature can be very large, as much as 100 per cent or more, as compared with the first order theory alone. This observation is again in agreement with the results of [12].

For $\pi_3 < -1{\cdot}0$, rather general statements of the above type cannot be made regarding the second order effects since there is a range of β in the neighborhood of 0·0 for which $h'_{3c}(0)$ can be negative. In addition, for large negative π_3, singularities appear in the solutions for $\beta > \beta_{sep}$ and it is doubtful whether the asymptotic solutions are uniformly valid, since the second order

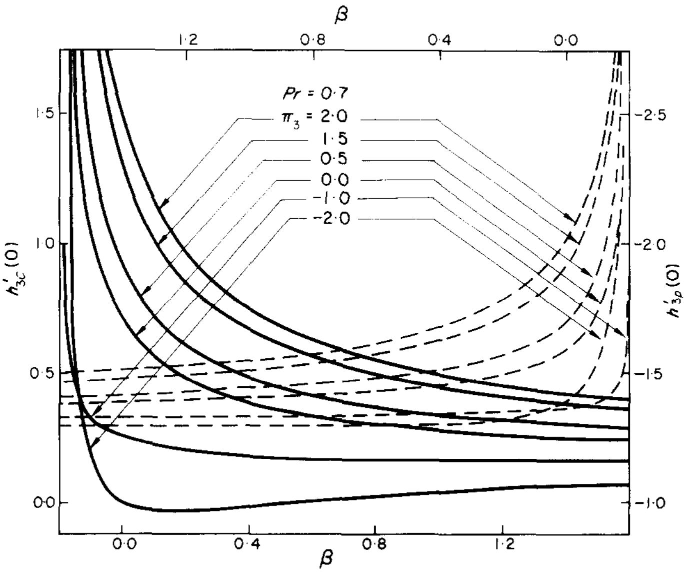

FIG. 7. Plot of $h'_{3c}(0)$ and $h'_{3p}(0)$ against β, $Pr = 0{\cdot}7$.

quantities seem to become an order of magnitude greater than the first order boundary layer quantities themselves.

The solutions for transverse curvature are exhibited in Figs. 9 and 10. The qualitative aspects of these solutions are similar to those for longitudinal surface curvature, as may be readily expected from the similarity in the second order equations for longitudinal and transverse curvatures. The second order contribution to heat transfer is again a minimum in magnitude when the pressure gradient is favorable. The gradients become singular when $\beta = \beta_{sep}$ is approached. The singularities in $h'_{4c}(0)$ for all

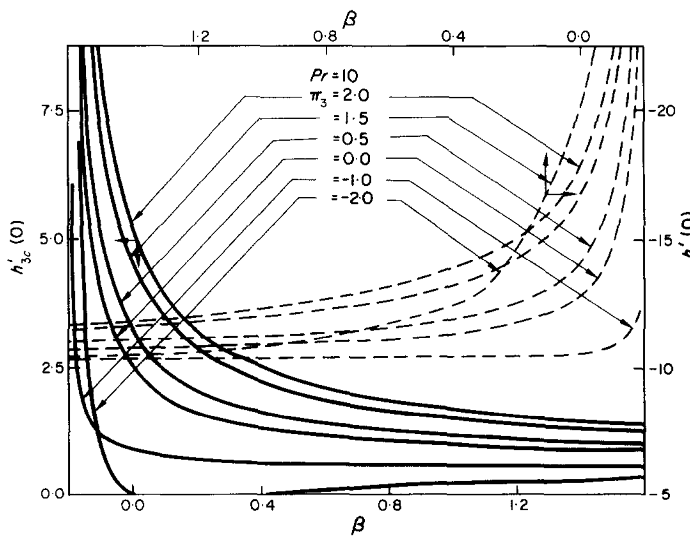

FIG. 8. Plot of $h'_{3c}(0)$ and $h'_{3p}(0)$ against β, $Pr = 10$.

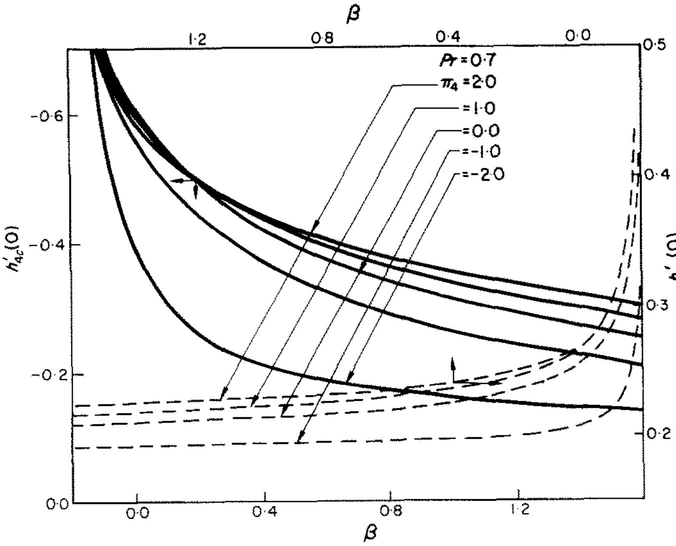

FIG. 9. Plot of $h'_{4c}(0)$ and $h'_{4p}(0)$ against β, $Pr = 0{\cdot}7$.

$\pi_4 > -0{\cdot}5$ can be approximated by the relations

$$h'_{4c}(0) \propto (\beta - \beta_{sep})^{-0{\cdot}3}, \quad Pr = 0{\cdot}7$$

$$h'_{4c}(0) \propto (\beta - \beta_{sep})^{-0{\cdot}48}, \quad Pr = 10.$$

The major difference between the longitudinal and transverse curvature effects is in the sign of the second order gradients. While $h'_{3c}(0)$ is positive and $h'_{3p}(0)$ is negative for the most part, $h'_{4c}(0)$ is negative and $h'_{4p}(0)$ is positive for all $\pi_4 > -1{\cdot}0$. Thus longitudinal and transverse curvatures have opposing effects on heat transfer for similar curvatures. It may therefore be possible to off-set the effect of longitudinal curvature by a suitable choice of

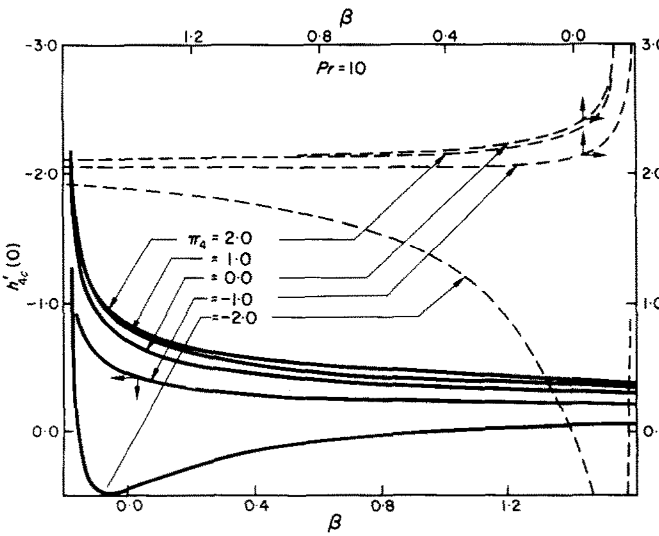

FIG. 10. Plot of $h'_{4c}(0)$ and $h'_{4p}(0)$ against β, $Pr = 10$

transverse curvature for any given pressure gradient, as far as heat transfer to the second order is concerned.

By choosing the surface curvatures properly, it is possible to make the wall adiabatic, correct to the second order. This occurs if Nu in equation (25) is equated to zero when there is obtained

$$\frac{\varepsilon\sqrt{(2\xi)}}{U_1}\left[\frac{B(0)}{U_1}h_1'(0) + \frac{U_2}{\sqrt{(2\xi)}}h_2'(0) + \frac{k}{r^j}h_3'(0) + \frac{j\cos\omega}{r^{2j}}\; h_4'(0)\right]_{\text{adiabatic}} + \theta'(0) = 0. \quad (27)$$

The equation, in general, specifies only certain points along the surface at which adiabatic conditions exist. However, for a flow with no free stream vorticity and where conditions of joint similarity exist (i.e. $\pi_2 = \beta, \pi_3 = \pi_4 = \beta - 1$), the above equation simplifies to

$$\varepsilon[C_d h_2(0) + C_l h_3'(0) + C_t h_4'(0)] + \theta'(0) = 0 \quad (28)$$

where C_d, C_l and C_t are given by the equations $C_d = U_2/U_1$, $C_l = k\sqrt{(2\xi)}/(U_1 r^j)$ and $C_t = j\cos\omega\sqrt{(2\xi)}/(U_1 r^{2j})$. Using equation (28), if one knows C_d and C_t, the magnitude of C_l that reduces the wall to adiabatic conditions may be determined for jointly similar situations. For positive π_2 and π_4, since $h_2''(0)$ and $h_4''(0)$ are negative, C_l will be positive when the Eckert number is zero or is small. When the Eckert number rises, it is possible for C_l to become negative. This means that the curvature changes from convex to concave with increasing Eckert numbers for an adiabatic wall.

Concluding, it is seen that free-stream vorticity, displacement speed, longitudinal curvature and transverse curvature affect heat transfer considerably, and the second order effects should not, in general, be neglected. The wall heat transfer effects vary considerably with Eckert number, especially close to separation.

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EINFLÜSSE DER VERWIRBELUNG, DER VERDRÄNGUNGSGESCHWINDIGKEIT UND DER KRÜMMUNG AUF DEN WÄRMEÜBERGANG BEI DISSIPATION

Zusammenfassung— Die Methode der angepassten asymptotischen Entwicklungen wurde benutzt, um den Wärmeübergang mit Dissipation bei Zwangskonvektion in inkompressibler laminarer Strömung an einer gekrümmten Oberfläche zu untersuchen. Mit Hilfe der bekannten Lösungen für die Falkner-Skan-Keilströmung und mit Hilfe der zugehörigen Energiegleichung mit Dissipationsglied als Effekt erster Ordnung wurden die Lösungen zweiter Ordnung für die Energiegleichung mit Dissipation ermittelt. Die berechneten Temperaturprofile zweiter Ordnung wurden graphisch gezeigt als Funktionen des Druckgradienten und der Prandtl-Zahl und, um die Auswirkungen der Verwirbelung des Freistromes zu zeigen, der Verdrängungsgeschwindigkeit und der Krümmungen in Längs- und Querrichtung. Der Wärmeübergang bei Berücksichtigung der Effekte zweiter Ordnung wird, ähnlich der Wandreibung bei Berücksichtigung der Effekte zweiter Ordnung, singulär bei gegenläufigen Druckgradienten, was bei erster Ordnung Ablösung entspricht. Weiterhin ergeben sich für das Problem der Verdrängungsgeschwindigkeit und für Probleme gekrümmter Oberflächen weitere Singularitäten bei kritischen Werten des Druckgradienten, ebenso wie bei der Wandreibung. Wenn keine Singularitäten auftreten, kann die Dissipation den Wärmeübergang beachtlich beeinflussen, indem die Nusselt-Zahl von positiv auf negativ umschlagen kann, in Abhängigkeit von der Grösse der Eckert-Zahl. Ein Anwachsen der Prandtl-Zahl verstärkt den Einfluss der Dissipation.

EFFETS DE LA VORTICITÉ, DE LA VITESSE ET DE LA COURBURE SUR LE TRANSFERT THERMIQUE AVEC DISSIPATION.

Résumé—La méthode des développements asymptotiques a été utilisée pour analyser le transfert thermique avec dissipation en convection forcée par un écoulement laminaire incompressible sur une surface courbe. En utilisant les solutions classiques de Falkner–Skan pour l'écoulement sur un dièdre et l'équation d'énergie correspondante sans dissipation comme effets du premier ordre, les solutions de second ordre ont été obtenues pour l'équation d'énergie avec dissipation. Le profil de température calculé au second ordre a été présenté graphiquement en fonction du paramètre de gradient de pression et du nombre de Prandtl pour montrer les effets de la vorticité de l'écoulement libre, de la vitesse, des courbures longitudinales et transversales. Le transfert thermique au second ordre, comme le frottement pariétal au second ordre devient singulier pour les gradients de pression adverse près du point de séparation du premier ordre. Pour le problème de la vitesse de déplacement et les problèmes de coubure d'autres singularités se rencontrent aux valeurs critiques du paramètre de gradient de pression juste comme le frottement pariétal du second ordre. Quand il n'y a pas de singularité la dissipation peut affecter considérablement le transfert thermique, changeant le nombre de Nusselt des valeurs positives en valeurs négatives, en dépendance de la valeur du nombre d'Eckert. Un accroissement du nombre renforce les effets de la dissipation.

ВЛИЯНИЕ ЗАВИХРЕННОСТИ, СКОРОСТИ ВЫТЕСНЕНИЯ И КРИВИЗНЫ НА ТЕПЛООБМЕН ПРИ НАЛИЧИИ ДИССИПАЦИИ

Аннотация—Используется метод подобранных асимптотических разложений для анализа теплообмена при наличии диссипации в условиях вынужденной конвекции в несжимаемом ламинарном потоке на искривленной поверхности. Применяя хорошо известные решения Фокнера-Скэна для обтекания клина и соответствующее уравнение энергии с учётом диссипации в качестве эффектов первого порядка, получены решения второго порядка уравнения энергии с учётом диссипации. Рассчитанные температурные поофили второго порядка в виде функции от параметра градиента давления и числа Прандтля представлены графически с тем, чтобы показать эфекты завихренности свободного потока, скорости вытеснения, продольной и поперечной кривизны. Теплообмен второго порядка, как и поверхностное трение второго порядка, становится необычным при наличии отрицательных градиентов давления вблизи точки отрыва, определяемой в первом приближении. Далее, в задачах о скорости вытеснения и кривизне имеют место другие отклонения от обычной картины при критических значениях параметра градиента давления как и для поверхностного трения во втором порядке. При отсутствии отклонений диссипация может значительно влиять на теплообмен; при этом число Нуссельта изменяется от положительного до отрицательного в зависимости от величины числа Эккерта. Увеличение числа Прандтля приводит к увеличению влияния диссипации.